

# Hysteresis mediated by a domain wall motion

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## Abstract

The position of an interface (domain wall) in a medium with random pinning defects is not determined unambiguously by a current value of the driving force even in average. Based on general theory of the interface motion in a random medium we study this hysteresis, different possible shapes of domain walls and dynamical phase transitions between them. Several principal characteristics of the hysteresis, including the coercive force and the curves of dynamical phase transitions obey scaling laws and display a critical behavior in a vicinity of the mobility threshold. At finite temperature the threshold is smeared and a new range of thermally activated hysteresis appears. At a finite frequency of the driving force there exists a range of the non-adiabatic regime, in which not only the position, but also the average velocity of the domain wall displays hysteresis.

*Key words:*

*PACS:*

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## 1 Introcuccion

This is our tribute to the memory of Per Bak, great scientist and an exceptional personality. His main contribution to science is the discovery of a new, very broad class of nonlinear deterministic systems displaying stable chaotic critical behavior, which he called Self-Organized Criticality (SOC). It is difficult to overestimate the significance of this discovery, which has opened new

ways for explanation and unified description of very remote phenomena, such as earthquakes, plastic deformations, hysteresis in magnets, dynamics of biosphere and financial catastrophes. The introduced by Per notion of avalanches is now central for dynamics of such systems and is most probably a main source of the  $1/f$  noise. Theory of these systems has deserved a wide popularity and Per is one of the most cited physicists.

However, his moral influence in the scientific community was not less important than his scientific contribution. He always was surrounded by people seeking new ways in different fields: geologists, biologists, physicians, economists, historians and, certainly physicists. Everybody felt himself free in this community to criticize everything and to propose new ideas, but Per's authority and respect to him was extremely high. It was based on his uncompromising pursuing of truth and hate to any falsehood in science as well as in social and personal life. Once he was shown by a big TV company at JFK airport expressing his indignation by the violation of passenger rights. He was a brave and noble man. He could be very harsh in discussions, a feature, which not everybody could tolerate. However, Per was a reliable, friendly and responsive person ready to help if there was a need in his help, and for this purpose he could spend his time, money and influence. We feel that his death has left a vacancy in our community, which will not be filled.

In this review we consider a simple, but rather wide-spread phenomenon: the hysteresis mediated by a motion of an interface or domain wall (DW) driven by an external alternating force in a medium with random pinning centers. The interplay between the driving force and elastic and pinning forces develops in time. If the driving force is constant, it can establish an average velocity of the DW after a sufficiently long time interval. In the case of the alternating force this time may be longer or shorter than period of oscillations resulting in different regimes. The influence of a heat reservoir gives an additional dimension to this phenomenon. The interface is a typical system displaying the SOC[1]: in the absence of the external force it exhibits a fractional structure. A small force results in avalanches on spatial and temporal scales depending on the force. The stronger is the force and the higher is the frequency the weaker is the influence of the avalanches, but they are crucial for the range of small frequencies and amplitudes important in the experiment.

This problem is a part of a very old hysteresis problem [2] occurring in many different dynamical processes: chemical reactions, magnetization reversal, crystal growth, absorption and desorption etc., in which many DW are involved. In this case the interaction between walls must be taken in account. It is beyond the scope of our consideration, but some rough estimates can be done on the basis of one-DW theory. On the other hand, the recent experimental development with its tendency to sub-micron spatial scale generated many small systems in which the motion of one or few DW is absolutely realistic. Moreover, the motion of an individual domain wall was experimentally discovered and studied by several groups of experimenters [3]. Therefore, theory

of such a motion is necessary for understanding of the observed phenomena and prediction of new ones.

Though the observation of the hysteresis in the ferromagnets accounts more than 100 years history, the phenomenon could not be satisfactory explained until the end of 20-th century, when a theory of DW and its motion in a random medium was developed. Therefore we start our review with an introductory section briefly describing this theory. In the next section we consider a DW moving adiabatically at zero temperature. Adiabatic motion means that the frequency of the driving force is small enough and the momentarily velocity is equal to its stationary value at a constant force equal to the momentarily value of the ac force. The third section is dedicated to the influence of finite temperature, but still in adiabatic regime. In the fourth section we consider non-adiabatic regime predict a new phenomenon: the hysteresis of velocity (magnetization rate in the case of a ferromagnet). In the last section we review relevant experiments.

## 2 Domain wall in a random medium

### 2.1 Zero temperature

For definiteness we consider a DW in an impure ferromagnet with uniaxial anisotropy at zero temperature. As it was shown in [4][5][6][7], equation of motion for a domain wall without overhangs can be written in a following way

$$\frac{1}{\gamma} \frac{\partial Z}{\partial t} = \Gamma \nabla^2 Z + h + \eta(\mathbf{x}, Z) \quad (1)$$

where  $Z(\mathbf{x}, t)$  denotes the interface position;  $\gamma$  and  $\Gamma$  are the domain wall mobility and stiffness, respectively;  $h$  is the external driving force. For a ferromagnet  $h = \mu_B H M$ , where  $H$  is the external magnetic field and  $M$  is the magnetization. Finally  $\eta$  denotes the random force generated by the impurities. For broad domain walls  $\Gamma \approx J/(a^{D-1}l)$ , where  $D$  denotes the dimensionality of the wall. For narrow walls  $\Gamma$  depends in general on  $J$ ,  $T$ , and the disorder strength in a complicated way [8].

The random forces  $\eta(\mathbf{x}, Z)$  generated by pinning centers obey the Gaussian statistics and have short range correlations:

$$\overline{\eta(\mathbf{x}, Z)\eta(\mathbf{x}', Z')} = \eta^2 l^{D+1} \delta_l(\mathbf{x} - \mathbf{x}') \Delta(Z - Z'). \quad (2)$$

Here  $\delta_l(\mathbf{x})$  denotes a delta-function smeared out over a distance  $l$ . The initial correlator  $\Delta_0(Z)$  is an even analytical function of  $Z$ , which decays to zero

over a finite distance  $l$  and has maximum at  $z = 0$ .

In the following we assume that the disorder is weak, i.e. that  $\Gamma \gg \eta l$ . Under this assumption the interface is essentially flat on length scales  $L \ll L_p$  (see [6]), where  $L_p \approx l \left( \frac{\Gamma}{\eta l} \right)^{2/(4-D)} \gg l$  is the so-called Larkin length. On larger scales the wall adapts to the disorder and gets pinned for driving fields  $h \lesssim h_p$  with  $h_p \approx \Gamma L_c^{-2} = \eta \left( \frac{\eta l}{\Gamma} \right)^{D/(4-D)} \ll \eta$  for the pinning threshold. The transverse displacement  $w(L)$  of the DW at the scale  $L$  is determined by the roughness exponent  $\zeta$ :  $w(L) \propto L^\zeta$  ( $\zeta \leq 1$ ). The characteristic time of an avalanche  $t(L)$  at the scale  $L$  is determined by the dynamic exponent  $z$ :  $t(L) \propto L^z$ .

If  $h$  exceeds  $h_p$ , the wall starts to move. Close to the depinning transition the velocity, which can be considered as an order parameter of the transition, vanishes according to a power law

$$v \approx v_p \left( \frac{h - h_p}{h_c} \right)^\beta, \quad h > h_p. \quad (3)$$

Here we introduced the characteristic velocity scale  $v_p = \gamma h_p$ . The moving domain wall conserves the rough (fractal) structure at length scales between  $L_p$  and a new scale established by small velocity  $\xi_v = L_p (v_p/v)^{\frac{1}{z-\zeta}}$ , where the pinning force dominates. At larger scales the bumps on the DW are healed and it becomes smooth.

The functional renormalization group calculations [6][7][9] show that  $1 < z < 2$ , i.e. the dynamics close to the depinning transition is *super-diffusive*, reflecting the rapid motion of the object after the maximum of the potential has been overcome. The critical exponents satisfy new scaling relations [6]

$$\nu = \frac{1}{2-\zeta} = \frac{\beta}{z-\zeta} \geq \frac{2}{D+\zeta}. \quad (4)$$

These exponents were calculated first to order  $\epsilon = (4-D)$  in [6] and recently to order  $\epsilon^2$  [9]:  $\zeta = \frac{\epsilon}{3}(1 + 0.14331\epsilon)$  and  $z = 2 - \frac{2\epsilon}{9} - 0.04321\epsilon^2$  [9].

In the opposite regime  $L \gg \xi$  the problem is essentially linear and  $Z$  can be replaced by  $vt$  in the argument of  $\eta(\mathbf{x}, Z)$ . This can be seen qualitatively as follows: On the time scale  $t$  the domain wall advances on average by an amount  $vt$ . Randomly distributed pinning centers will lead to a local distortion which, according to dynamical scaling, spreads over a region  $L(t) \approx L_p (\omega_p t)^{1/z}$ . The local retardation or advancement of the object due to the fluctuation in the density of the pinning centers scales as  $Z(t) \approx l(L(t)/L_p)^\zeta \approx l(\omega_p t)^{\zeta/z}$ , where  $\omega_p = v_p/l$  is the basic frequency at the Larkin length. Since  $\zeta < z$ ,  $Z(t)$  grows more slowly than  $vt$ . Thus on time scales  $t > t_v = \omega_p^{-1} (v_p/v)^{z/(z-\zeta)}$  and length scales  $L > \xi_v \equiv L(t_v)$  the non-linearities in the argument of  $\eta(\mathbf{x}, vt + Z)$  can be neglected and the linearized theory applies. In this case  $Z(\mathbf{x}, t)$  is replaced

by  $vt$  in the argument of the random forces. Random forces act then as a thermal noise with the temperature  $\sim v^{-1}$ .

After integrating out the interface fluctuations on the length scales  $L \lesssim \xi_v$ , the effective equation of motion for the interface profile  $Z(\mathbf{x}, t) = \langle Z(\mathbf{x}, t) \rangle_{\xi_v, t_v}$  where  $\langle \rangle_{\xi_v, t_v}$  denotes the spatial and time average over scales  $\xi_v$  and  $t_v$ , respectively, on large scales coincides formally with (1) with some values substituted by their renormalized counterparts:  $\gamma \rightarrow \gamma_{\text{eff}} = \gamma \left( \frac{\xi_v}{L_p} \right)^{2-z}$  and  $h \rightarrow h - h_p$ . The influence of the random field on these length scales is negligible.

## 2.2 Thermally activated creep of domain walls

Next we consider the influence of thermal noise on the interface motion. Thermal fluctuations lead to a no-zero velocity of the domain wall even at small forces. All changes in  $h$  are still assumed to be made *adiabatically*. The right-hand side of equation of motion (1) must be complemented by *thermal noise*  $\zeta(\mathbf{x}, Z, t)$  normalized as usual:

$$\langle \eta(\mathbf{x}, Z, t) \eta(\mathbf{x}', Z', t') \rangle_T = 2 \frac{T}{\gamma} \delta(\mathbf{x} - \mathbf{x}') \delta(Z - Z') \delta(t - t'). \quad (5)$$

In the absence of external driving force the typical free energy fluctuations of the length scale  $L$  are of the order  $F(L) = T_p (L/L_p)^{\tilde{\chi}}$  where  $\tilde{\chi} = D - 2 + 2\tilde{\zeta}$ . The *energy barriers* between different metastable states scale have the same order of magnitude. When the driving force  $h$  switches on, it changes barriers between neighboring metastable states at the scale  $L$  by the value  $-hw(L) = -hL_p(L/L_p)^{\zeta}$ . Thus, the total energy barrier is:

$$E_B(L, h) \approx F(L) - hL^D w_R(L) = T_p \frac{L^{\tilde{\chi}}}{L_p^{\tilde{\chi}}} \left( 1 - \left( \frac{L}{L_h} \right)^{2-\tilde{\zeta}} \right), \quad (6)$$

where we introduced the force length scale  $L_h = L_p \left( \frac{h_p}{h} \right)^{1/(2-\tilde{\zeta})}$ . A schematic graph of  $E_B(L, h)$  vs.  $h$  is shown in Figure 1, it has a maximum at  $L = \tilde{L}_h \sim L_h$  and vanishes for  $L = L_h$ . The maximal height of the barrier is:

$$\tilde{E}_B(h) \equiv E_B(\tilde{L}_h, h) \approx T_p \left( \frac{h_p}{h} \right)^{\mu}, \quad \mu = \frac{\tilde{\chi}}{2 - \tilde{\zeta}}. \quad (7)$$

It must be overcome by thermally activated hopping to initiate motion. The *creep velocity* of the domain wall follows from  $v_{\text{creep}} \approx w(\tilde{L}_h)/\tau(\tilde{L}_h)$ . According to the Arrhenius law, the hopping time is  $\tau \sim \omega_p^{-1} e^{\tilde{E}_B(h)/T}$ . Thus, we obtain

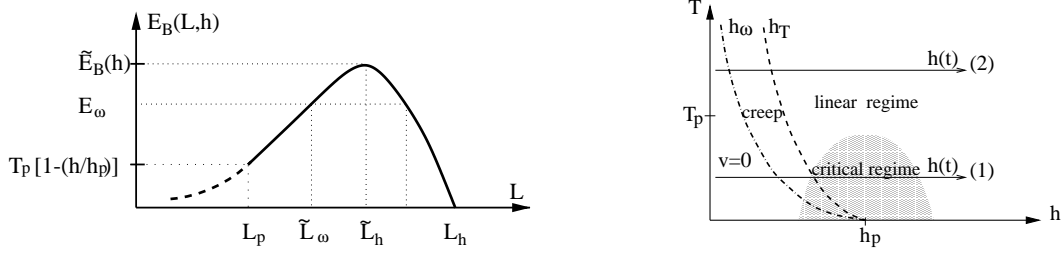


Fig. 1. *Left*: Energy barrier as a function of the length scale  $l$  for a given driving force density  $h$ . *Right*: The lines  $h_\omega$  and  $h_T$  as explained in the text.

$v_{\text{creep}}(h) \approx \frac{w(\tilde{L}_h)}{\tau(\tilde{L}_h)} \sim \exp\left[-\frac{T_p}{T} \left(\frac{h_p}{h}\right)^\mu\right]$ . We have omitted a prefactor, which is beyond our accuracy. This formula is valid for  $T \ll \tilde{E}_B(h)$  and was found first by Ioffe and Vinokur[15] (see also [16]). In the opposite case  $T \gg \tilde{E}_B(h)$  we expect a linear relation between the driving force and the velocity:  $v \simeq \gamma h$ . The border line between the two cases i.e. the inflection point of the curve  $v(h)$ ,  $T \approx \tilde{E}_B(h)$ , defines a *temperature dependent force*  $h_T$  (compare Figure 1.)

$$h_T = h_p \left(\frac{T_p}{T}\right)^{1/\mu}. \quad (8)$$

Note that the creep formula is valid only for  $h \ll h_T$ .

Let us now consider the influence of thermal fluctuations on the depinning transition. At  $h \leq h_p$  and  $T = 0$  the velocity is zero, but one has to expect that as soon as thermal fluctuations are switched on, the velocity will become finite. Scaling theory predicts in this case an Ansatz [18,19] (generalizing (3))

$$v(h, T) \sim T^{\beta/\tau} \Phi\left(\frac{h - h_p}{T^{1/\tau}}\right) \quad (9)$$

with  $\Phi(x) \rightarrow \text{const.}$  for  $x \rightarrow 0$  and  $\Phi(x) \sim x^{\beta/\tau}$  for  $x \gg 1$ , such that  $v(h_p, T) \sim T^{\beta/\tau}$ .  $\tau > 0$  is a new exponent which still has to be determined.

It is worthwhile to note that relevant thermal fluctuations, which unpin the domain wall act on scales of the order of  $L_p \ll \xi$  as was first indicated by A. Middleton [19]. At the critical point  $h = h_p$  essentially only barriers on the scale  $L \approx L_p$  are left as we saw earlier. It is therefore sufficient to consider only this length scale. A detailed analysis of the form of the effective potential on this scale gives  $\tau = 3/2$ .

### 3 Adiabatic motion of a single domain wall driven by ac force

#### 3.1 Dynamics of a rectilinear domain wall at zero temperature

In this section we describe the motion of a DW, rectilinear at large scale [20]. The reason of the motion is the alternating driving force, which we assume to have a simple harmonic shape:  $h(t) = h_0 \sin \omega t$ . In previous section we demonstrated that the DW roughness can be ignored on a time scale  $t > t_v$  and the length scale  $L > L_v$ . Thus, the bending of the DW can be neglected if  $\omega t_v \ll 1$ . Simultaneously this condition means that the process is adiabatic, i.e. that the velocity at any moment of time with high precision is equal to its stationary value corresponding to the value of the driving force  $h(t)$  at the same moment of time. The rectilinear (or plane) DW can be characterized by one coordinate only. We denote it  $Z(t)$ . In adiabatic approximation it satisfies an obvious equation:

$$\frac{dZ}{dt} = \gamma h_p f \left( \frac{h(t) - h_p}{h_p} \right), \quad (10)$$

where the scaling function  $f(x)$  behaves asymptotically as  $x^\beta$  at small  $x$  and as  $x$  at large  $x$ . Instead of integrating it over time, we integrate it over field by the following change of coordinate:  $dt = \frac{dh}{\omega \sqrt{h_0^2 - h^2}}$ . Thus, we find a following expression for  $Z$  as function of the field  $h$ :

$$Z(h, h_0, Z_0) = Z_0 + \frac{\gamma h_p}{\omega} \int_{h_p}^h f \left( \frac{h' - h_p}{h_p} \right) \frac{dh'}{\sqrt{h_0^2 - h'^2}} \quad (11)$$

This equation is correct for  $h > h_p$ ; at smaller positive values of  $h$  the DW does not move and  $Z = Z_0 = \text{const}$ . Therefore, the magnetization remains constant until the amplitude of the driving field reaches the value  $h_p$ . This value marks the first dynamical phase transition: the appearance of the hysteresis loop. Now starting from  $Z_0 = -L$  at  $h = 0$ , where  $L$  is the half of size of a rectangular sample, and applying positive magnetic field, one can increase  $Z$  until it takes its maximum value

$$Z_{\max} = -L + \frac{2\gamma h_p}{\omega} \int_{h_p}^{h_0} f \left( \frac{h' - h_p}{h_p} \right) \frac{dh'}{\sqrt{h_0^2 - h'^2}} \quad (12)$$

This equation is valid as long as the right-hand side of equation (12) is less than  $L$ . When it becomes larger, the maximal value of  $Z$  obviously remains

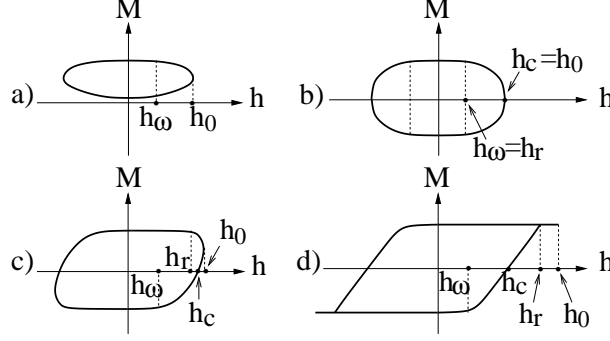


Fig. 2. Shapes of the hysteresis loops and dynamical phase transitions

equal to  $L$ . The value  $h_{t2}$  of the amplitude  $h_0$ , at which  $Z_{\max}$  becomes equal to  $L$  determines the second dynamical phase transition: the complete reversal of magnetization. The hysteresis loop becomes symmetric with respect to inversion:  $h \rightarrow -h$ ,  $M \rightarrow -M$  ( $Z \rightarrow -Z$ ) (see Figure 2b). At smaller values of  $h_0$  between  $h_p$  and  $h_{t1}$  the reversal of the magnetization is incomplete, the hysteresis loop is symmetric with respect to reflection  $h \rightarrow -h$ , but not to inversion of magnetization (see Figure 2a). Finally, when  $Z$  reaches the value  $L$  for a quarter of period or less, i.e. at  $h \leq h_0$ , in a range of fields  $h_r < h < h_0$  the magnetization reaches the saturation (in a little more realistic model it becomes a single-valued function of the field). We call these parts of the magnetization graph whiskers (see Figure 2c). The value  $h_{t3}$  of  $h_0$ , at which the whiskers first appear determines the third dynamical transition. It can be found from the obvious equation:  $Z(h_{t3}, h_{t3}, -L) = L$ , where we have used notations of equation (11). Note that similar equation is valid for  $h_{t2}$ :  $Z(h_{t2}, h_{t2}, -L) = L/2$ . From these equation we find a scaling relation for the phase transitions fields:

$$\frac{h_{t2}}{h_p} = F\left(\frac{\omega L}{2\gamma h_p}\right); \quad \frac{h_{t3}}{h_p} = F\left(\frac{\omega L}{\gamma h_p}\right), \quad (13)$$

where  $F(x)$  is a function inverse to  $G(y) = \int_1^y f(y' - 1) \frac{dy'}{\sqrt{y^2 - y'^2}}$ .

It is accepted to characterize the hysteresis loop by the coercive force  $h_c$ , i.e. by a value of the driving force at which the magnetization vanishes ( $Z = 0$ ):  $Z(h_c, h_0, -L) = 0$ . Earlier we have introduced the saturation field  $h_r$ . Equation for it is:  $Z(h_r, h_0, -L) = L$ . Equation (11) implies that these two fields divided by  $h_p$  obey the scaling equations depending on 2 dimensionless arguments  $u = \frac{\omega L}{\gamma h_p}$  and  $v = \frac{h_0}{h_p}$ . The reader can find details in the original article [20]. We should mention the asymptotic behavior at very big  $u$  or  $v$ :  $h_{t2} \approx \frac{\omega L}{2\gamma}$ ;  $h_{t3} \approx 2h_{t2}$ ;  $h_c \approx \sqrt{h_{t2}(2h_0 - h_{t2})}$ ;  $h_r \approx \sqrt{h_{t3}(2h_0 - h_{t3})}$ . The critical asymptotics near the mobility threshold ( $v - 1 \ll 1$ ) is more complicated. The details can be found in the same article [20].

Numerical MC simulation of the 2-dimensional Ising model with random bonds



and the Glauber dynamics [20] demonstrated that the DWs, initially numerous, quickly merge to a few ones with linear size of the same order as the size of the sample. This stage of the magnetization reversal is the longest and determines dynamics in total. Thus, our results are qualitatively correct for larger systems at the last and longest stage of the hysteresis process. The same numerical simulation has reproduced all types of the hysteresis loop predicted by theory of the single DW hysteresis. However, the quantitative details may be different. Moreover, since theory of multidomain hysteresis does not yet exist, there is no reliable estimate of the time at which only few domains remain.

### 3.2 Motion of a domain wall at finite temperature

As it was shown in subsection 2.2, the principal difference of the dynamics at finite temperature is that the DW can start to move at an arbitrarily weak driving force due to thermal activation processes. Thus, strictly speaking, there is no more mobility threshold for the constant driving force. At low temperature the threshold will be smeared. Besides the finite temperature establishes new scales of the length  $L_T$ , time  $t_T \propto L_T^z$ , force  $h_T$  and activation energy  $E_B(T, h)$ . In adiabatic regime it leads to appearance of an effective, temperature dependent threshold for the ac driving force. In this section we follow the work [21]

If the driving force is alternating in time with frequency  $\omega$  and amplitude  $h_0$ , the barriers at the scale  $L$ , for which  $\omega\tau(L, h) > 1$ , where  $\tau(L, h) = \tau_0 \exp E_B(L, h)/T$  is the relaxation time for such a fluctuation, can not be overcome during one cycle of oscillation. Thus, the global motion of the DW may be initiated only when the maximum of value  $\omega\tau(L, h)$  over  $L$  becomes of the order of one. From the condition  $\omega\tau_{\max}(h) = 1$  we find a new, frequency and temperature dependent driving field  $h_\omega$ , which plays the role of dynamic threshold. It obeys a following equation:

$$\frac{h_\omega}{h_p} = \left[ \frac{T_p}{T\Lambda} \left( 1 - \frac{h_\omega}{h_p} \right)^{1/\mu} \right], \quad (14)$$

where  $\Lambda = -\ln(\omega\tau_0)$  (reminder:  $\tau_0 = \omega_0^{-1}$  is a microscopic hopping time: we assume  $\omega\tau_0 \ll 1$ ). At  $h < h_\omega$ , there is no macroscopic motion of the wall; though still its segments transfer between different metastable states with avalanches whose development time is less than  $2\pi/\omega$  and the length scale is less than  $\tilde{L}_{\omega} = L_p \left( \frac{T}{T_p} \ln \frac{\omega_p}{\omega} \right)^{1/\tilde{\chi}}$ .. This process gives rise to dissipation. Drift of the wall as a whole starts at  $h_0 > h_\omega$ . At  $\omega\tau_0 \ll 1$ ,  $h_\omega < h_T$ . Note that equation (14) determines  $h_\omega$  as a monotonously decreasing function of temperature accepting the value  $h_p$  at  $T = 0$ . At a fixed temperature  $h_\omega$  is

a monotonously decreasing function of frequency. Thus, DW subject to the ac driving force either remains in rest at  $h_0 < h_\omega$ , or moves due to thermal activation (creep regime) at  $h_\omega < h_0 < h_T$ , or it moves in the sliding regime at  $h > h_T$ . At low temperatures these three regimes overlap with the critical behavior near  $h = h_p$ . A schematic phase diagram of the DW motion in variables  $T - h$  is shown on the r.h.s of Figure 1.

Having derived the velocity of DW motion as a function of the driving field at fixed temperature and frequency, we can follow the hysteresis curve using the adiabatic equation of motion similar to that at zero temperature (10), but containing new parameters  $T$  and  $h_\omega$ :

$$\frac{dZ}{dh} \equiv v(h) = \frac{\gamma h}{\omega \sqrt{h_0^2 - h^2}} f\left(\frac{h}{h_\omega}, \frac{T}{T_p}\right), \quad (15)$$

where  $f(x, y)$  is a dimensionless function of dimensionless arguments which is equal zero at  $x < 0$  and 1 at  $x$  or  $y$  infinite.

The shape of hysteresis loops at finite temperature is rather similar to their shape at zero temperature. In particular, all 3 dynamic phase transitions described in subsection 3.1 proceed at finite temperature as well, but the role of the threshold driving force plays the field  $h_\omega$ . At the first transition the amplitude  $h_0$  coincides with  $h_\omega$ . At the second and the third transition amplitudes the same values are determined by equations:

$$\int_{h_\omega}^{h_{tn}} \frac{v(h)dh}{\sqrt{h_{tn}^2 - h^2}} = \frac{(n-1)\omega L}{2}; \quad n = 2, 3 \quad (16)$$

Equations (15,16) display the scaling similar to that at zero temperature: the dimensionless ratios  $h_{tn}/h_\omega$  are defined by the same function of dimensionless variable  $(n-1)\omega L/(2\gamma h_\omega)$ .

## 4 Non-adiabatic motion of a wall driven by an ac field

### 4.1 Zero temperature

We again consider the motion caused by an ac driving force  $h(t) = h_0 \sin(\omega t)$  in eq. (1) with a frequency  $\omega \ll \omega_p = \gamma h_p/l$ , but we will not assume adiabatic condition. We start with zero temperature.

The finite frequency  $\omega$  of the driving force acts as an infrared cutoff for the propagation of perturbations generated by pinning centers. As it follows from

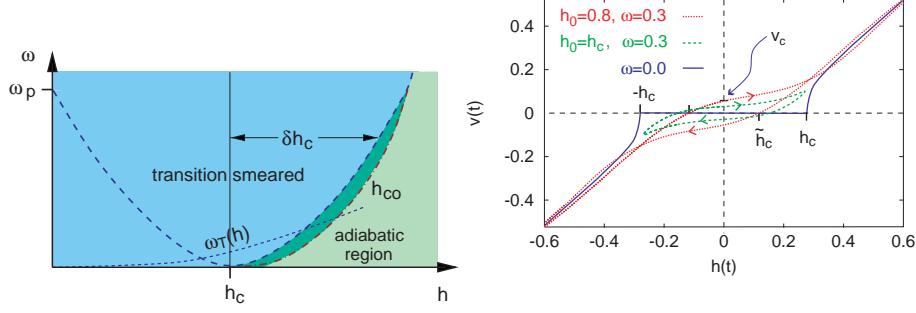


Fig. 3. *Left*: Schematic frequency-field diagram for the depinning in an ac external field (with  $h_0 > h_p$ ): For  $0 < \omega \ll \omega_p$  the depinning transition is smeared but traces of the  $\omega = 0$  transition are seen in the frequency dependency of the velocity at  $h = h_p$ . This feature disappears for  $\omega \gg \omega_p$ . *Right*: Velocity hysteresis of a  $D = 1$  dimensional interface in a random environment.

(1) these perturbations can propagate during one cycle of the external force up to the (renormalized) diffusion length  $L_\omega = L_p(\gamma\Gamma/\omega L_p^2)^{1/z} \equiv L_p(\omega_p/\omega)^{1/z}$ . If  $L_\omega < L_p$ , (i.e.  $\omega > \omega_p$ ) there is no renormalization and  $z$  has to be replaced by 2. During one cycle of the ac-drive, perturbations generated by local pinning centers affect the configuration of the domain wall only up to a scale  $L_\omega$ . If this scale is less than  $L_p$ , the resulting curvature force  $\Gamma/L_\omega^{-2}$  is always larger than the pinning force and there is no pinning anymore.

In the opposite case  $L_\omega > L_p$  (i.e.  $\omega < \omega_p$ ), the pinning forces compensate the curvature forces at length scales larger than  $L_p$ . As a result of the adaption of the domain wall to the disorder, pinning forces are renormalized. This renormalization is truncated at  $L_\omega$ . Contrary to the adiabatic limit  $\omega \rightarrow 0$ , there is no sharp depinning transition at  $\omega > 0$ . Indeed, a necessary condition for the existence of a sharp transition in the adiabatic case was the requirement that the fluctuations of the depinning threshold in a correlated volume of linear size  $\xi$ ,  $\delta h_p \approx h_p(L_p/\xi_v)^{(D+\zeta)/2}$ , are smaller than  $(h - h_p)$ , i.e.,  $(D + \zeta)\nu \geq 2$  [17]. For  $\omega > 0$  the correlated volume has a maximal size  $L_\omega$  and hence the fluctuations  $\delta h_p$  are given by

$$\delta h_p \approx h_p (L_p/L_\omega)^{(D+\zeta)/2} = h_p (\omega/\omega_p)^{(D+\zeta)/(2z)}. \quad (17)$$

Thus, different parts of the domain wall have different depinning thresholds – the depinning transition is *smeared* in the interval  $\delta h_p$ . For a better understanding of the velocity hysteresis we consider coupling between the different segments of the domain wall with the average lateral size  $L_\omega$ . Approaching the depinning transition from sufficiently large fields,  $h_0 \gg h_p$  (and  $\omega \ll \omega_p$ ), one observes the critical behavior of the adiabatic case as long as  $\xi_v \ll L_\omega$ . The equality  $\xi_v \approx L_\omega$  defines a field  $h_{co}$  signaling a crossover to an *inner* critical region where singularities are truncated by  $L_\omega$ . Note that  $h_{co} - h_p = h_p(\omega/\omega_p)^{1/(\nu\tilde{z})} \geq \delta h_p$  (cf. Figure 3).

A new physical phenomenon occurring in the non-adiabatic regime is the *hys-*

*teresis of velocity*. It is shown on the r.h.s. of Figure 3, which was obtained by numerical simulation of one-dimensional version of equation (1). The physical reason for this effect is the hampering of the DW by the pinning centers. It stops the average motion when the force becomes small, but not zero. Then the elastic forces tend to return too stretched intervals of the DW and submit it an average velocity opposite to the applied force, as is seen on the same figure. This consideration explains the clockwise circulation on the central hysteresis loop and appearance of secondary loops. The reader is referred for details can to reference [24].

## 5 Experiments

In this section we review several experimental works relevant to our topic. We start with the work by Budde *et al.* [25]. They studied the first order phase transition between a two-dimensional gas and two-dimensional solid in the first adsorbed monolayer of a noble gas (Ar, Kr, Xe) on the face (100) of NaCl. Adsorbed layer was in equilibrium with the 3d gas of the same atoms. Varying temperature linearly with time back and forth, they observed the hysteresis of the adsorbate density. The deviation of the temperature from transition point plays in this experiment the role of the driving force. They have found that the width of the hysteresis loop, which corresponds to the coercive force  $h_c$  in magnetic system, scales as  $(\Delta T)^{1/4}$ , where  $\Delta T$  is the amplitude of the temperature oscillations. This result agrees with theoretical predictions made in the work [20] (see the end of subsection 3.1). This result suggests that the experimental parameters corresponded to a regime of the force much larger than the threshold value, so that the pinning force was negligible.

In the works by Kleemann and coworkers [26,27] the authors studied the dielectric spectra in ferroelectric single crystals  $\text{Sr}_{0.61-x}\text{Ba}_{0.39}\text{Nb}_2\text{O}_6\text{Ce}_x^{3+}$  in the vicinity of its transition temperature. They have found different behavior of the electric susceptibility  $\tilde{\chi}$  vs. frequency. At low frequencies they observed a power-like behavior  $\chi \propto \omega^\gamma$  with  $\gamma$  between 0.2-0.7. Such a behavior the authors ascribe to the creep motion with a broad distribution of the activation energy. At higher frequencies they observed the logarithmic behavior of  $\tilde{\chi}$ , which they treat as the reversible relaxation of the DW segments. The crossover between these two regimes they attribute to the dynamical phase transition at  $h = h_\omega$  as it is predicted in the work [21] (see subsection 3.2).

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